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LAWS OF RAINFLOW

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WATERS WITHIN THE EARTH.

An abundant supply of fresh water is so essential to all the activities of life, that everywhere the question of rainfall is regarded with the keenest interest, and stations have been established throughout the world for keeping accurate records of the times and amounts of downpour. These observations, however, go no further than the surface of the ground. They tell us nothing about the subsequent history of the water as it journeys onward through the dark recesses of the earth! How much of it is taken up by evaporation? How much is needed to satisfy the demands of plant-life? Much less do they give the faintest idea of what quantity reappears in lake or stream after months of unseen flow?

Our research has the twofold object of supplying the missing history and developing the *laws of* subterranean flow, or, more concisely, *rainflow.* The standard for measure and comparison will be the household well, because it is found in every country and affords a ready access for underground study.

Let us trace the progress of a summer storm!—

The first drops that fall simply moisten the ground. Gradually the surface becomes saturated, after which water can no longer enter the soil as fast as the rain falls. The excess, therefore, must glide away over the surface to the lowlands. When the storm ceases a portion of the water will evaporate, but a larger portion will be taken up by vegetation. It is generally conceded among agricultural authorities that grasses and herbs require for perfect growth a *daily* supply of water *equal to their own weight.* Possessed of so great capacity, we can readily understand how the midsummer demand seizes all that escapes evaporation, and for the time being stops all further descent of the water. With the disappearance of vegetation during the winter months this demand ceases and the

water continues its descent as rapidly as the nature of the soil permits. Finally, it falls into what may properly be called the *great subterranean lake*, or, more concisely, the *sublake*. This body of *diffused water* underlies the earth's surface and is almost coextensive with its area. Its universal character is evidenced by the fact that wherever a well is sunk to a sufficient depth one is sure to find the sublake, and its water will fill the cavity only to the level of the sublake. We have described it as a body of diffused water, because its globules fill the interstices of the soil, sand, or disintegrated rock among whose particles it exists. Furthermore, it fills all fissures and openings in subjacent rock to which it has access. Most rocks are wholly impervious to water, but their extensive fissures store it in vast quantities. When, therefore, a well pierces one of these fissures an abundant supply is fully guaranteed. Evidently in sinking a well through solid rock the result is always a matter of chance, and depends upon whether the drill encounters a fissure large enough to give the needed supply without going to some extraordinary depth. Occasionally a hillside fissure is not perfectly enclosed, but has a minute opening at the surface of the ground. In such case the water bubbles forth as a refreshing spring. Artesian wells count their depth by hundreds of feet, and finally pierce water-bearing strata whose surface outcrop is likely to be found many miles distant. These storage strata are filled with water of the great sublake. When the lake surface stands at a greater elevation than the mouth of the well, the discharge takes place under pressure and the water rises like a fountain.

In making a comparison between the sublake and lakes found on the earth's surface, we observe:

1*st*. The sublake is not affected by storms.

2*d*. It has no tide.

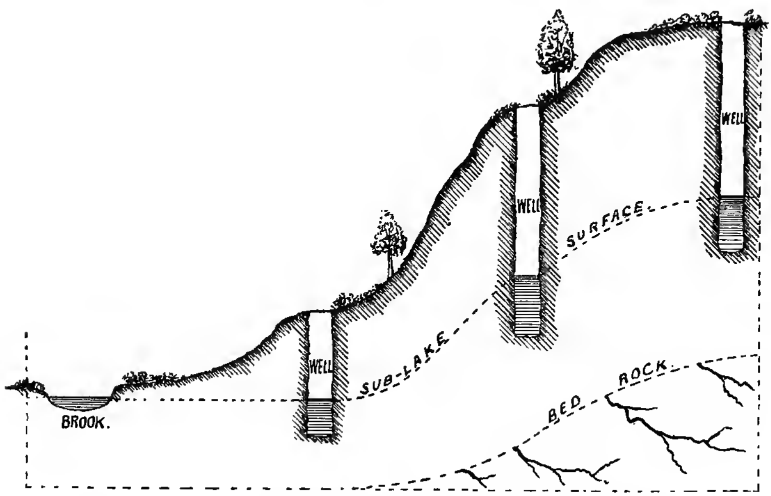
3*d*. Its surface is never frozen.

4*th*. It lacks mobility.

These characteristics are principally due to the mass of soil overlying the sublake, but the last feature is developed by the resistance of the particles around which the water is obliged to work its way. Hence the surface of the sublake is undulating in a hilly country. If all hills were composed of coarse gravel, all rainfalls entering the

earth would quickly find a common level. It is only due to the close texture of the soil that the water is *held back upon the hillsides*, and so little is lost, in the intervals between storms, by percolation, that it continues in a *banked-up state* from year to year, ever parting with what it receives, and ever receiving fresh supplies from above.

FIG. 1.



Wells that are dug in hillsides, as in Fig. 1, reveal the surface of the sublake far above the level of the brook in the adjoining valley, yet the brook owes its own permanence to the sublake, whose waters for many months descending through subterranean passages ultimately reach the level of the brook and maintain its flow in times of drought. For our study a hilltop well is preferable, and its lining should exclude all surface water for a depth of at least 10 feet below the curb.

Influx depends upon the presence or absence of rainfall, clouds, and fog, the activity of vegetation, and at times upon the frozen condition of the soil.

Efflux is the *overflow* of the sublake, and occurs at a depth far below the reach of local causes, where earth aud water are practically of one temperature throughout the entire year, where ice is

unknown, and the earth's covering of snow exerts no appreciable influence on the flow. In a word, efflux goes on *throughout the entire year*, holding its even course, and for a given locality ever flowing at one continuous speed.

As the sublake is the *resultant* of influx and efflux, its surface rises or falls as one or the other gains the mastery.

A simple experiment with a dish, a huge sponge, and a pitcher of water will give a clear idea of the principle of influx. The dish represents the impervious clay or bed-rock beneath the soil which entirely stops the descent of the water. The sponge represents the surface soil that is open and porous, or the disintegrated rock. The pitcher of water symbolizes the annual rainfall. If water is poured over the sponge its porous cells will for some time take up all we give and the dish will remain dry. But continuing to pour, the cells will at length become saturated, and further additions cannot be made, because the surplus will simply pass through the sponge and accumulate in the dish below. The accumulation in the lower part of the sponge is a true counterpart of the sublake.

We learn from this experiment that no water can reach the sublake unless complete saturation of the soil first takes place. That the rapidity of flow depends on the texture of the soil through which the water percolates. Also that its efflux in a given time is practically *constant*, because the lay of the land naturally limits the border of the sublake. This feature of constancy is a most important factor in the study of the fluctuations,—in fact, a key to the situation, for it renders intelligible movements that otherwise would seem obscure.

The Sublake.

The changes that are perpetually taking place in climatic conditions give little rest to the sublake. They in fact develop both Annual and Periodic fluctuations and cause the waters to rise and ebb with tide-like flow.

Annual fluctuations showing a range of 63 inches are graphically portrayed in Fig. 2, which gives sectional views of a single well, and shows the relative height of water for five different months.

The heights for intermediate times are expressed by the curved line joining these surfaces.

Fig. 2.

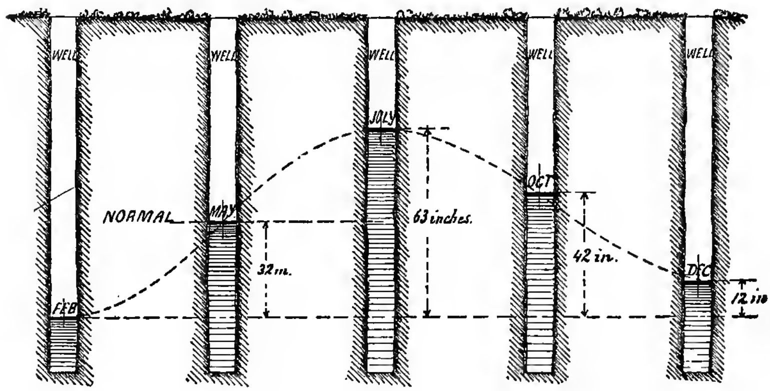
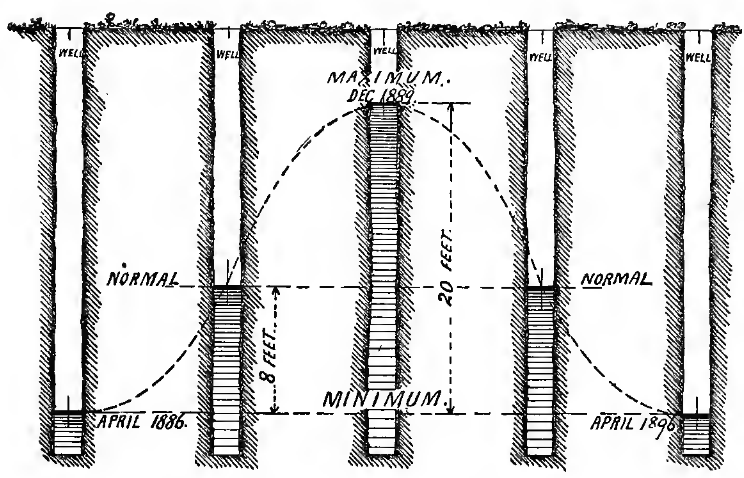


Fig. 3.



Periodic fluctuations showing a range of 20 feet are illustrated by Fig. 3. It should be noted that the maximum and minimum points are separated by years. In reaching these points the normal

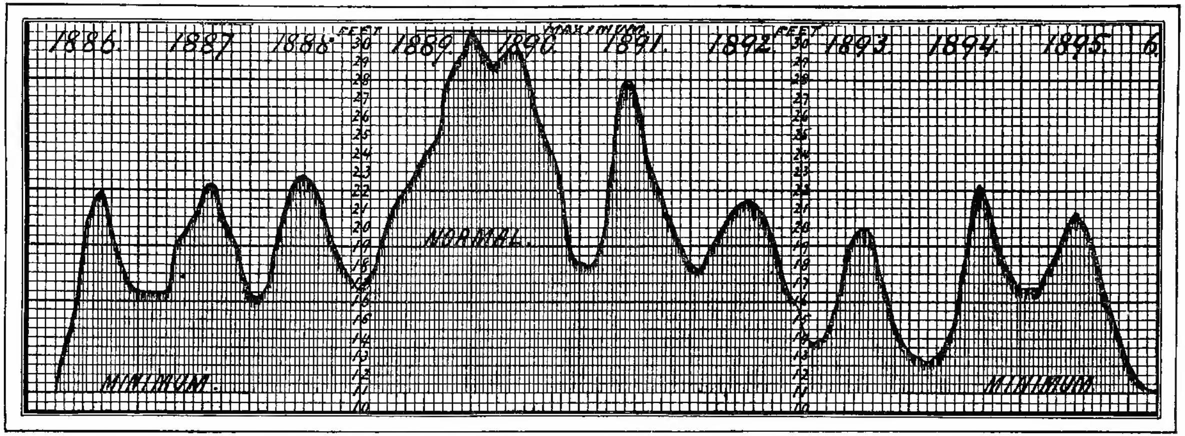
level may be crossed as often as twice in one year; still years occur in which the surface fails to pass the normal once.

During the past ten years a careful record has been kept of the water-level in Hill Crest well, Bryn Mawr, Pa. The readings have been tabulated plus (+) when influx prevailed, minus (—) when efflux, and (0.) when neither had the mastery. It will be observed that, beginning with exceptionally low water in April, 1886, the surface reached a maximum height December, 1889, and that after an interval of ten years the minimum was regained in April, 1896. The record, therefore, covers a cycle of events, and comprises a fully rounded period.

TABLE I.—MOVEMENT OF SUBLAKE IN HILL CREST WELL.

Year.	Jan.	Feb.	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Totals.
1886				27″	29″	37″	30″	9″	–23″	–21″	–14″	–8″	66″
1887	0″	–3″	0″	33	6	5	21	8	–19	–10	–22	–19	0
1888	–3	7	26	24	18	2	–8	–18	–20	–15	–8	–2	3
1889	9	12	24	12	8	9	13	7	34	16	8	15	167
1890	–8	–9	–7	10	8	–15	–23	–18	–18	–30	–30	9	–131
1891	–21	5	31	51	33	–6	–28	–24	–20	–13	–17	–12	–21
1892	–5	11	10	11	11	3	–2	–10	–17	–20	–15	–13	–36
1893	–15	0	9	18	26	18	4	–16	–20	–21	–12	–9	–18
1894	–4	–4	7	7	13	38	41	7	–24	–18	–17	–7	39
1895	0	0	13	10	16	10	–6	–14	–21	–19	–20	–14	–45
1896	–12	–9	–3										–24
Monthly averages,	–5″.9	1″.0	11″.0	20″.3	16″.8	10″.1	4″.2	–6″.9	–14″.8	–15″.1	–14″.7	–6″.0	Sums Equal

The monthly averages give us the general trend of events. Commencing with the month of February we find the sublake rose 1 inch. By the end of March it had risen 1 + 11 = 12 inches. By



SUBLAKE AT HILL CREST,

SHOWING THE RISE AND FALL OF ITS SURFACE DURING TEN YEARS ENDING 1896.

the end of April the elevation amounted to $1 + 11 + 20.3 = 32.3$ inches, and so on for the remainder of the year. Gathering these quantities, we have—

TABLE II.—AVERAGE RISE OF HILL CREST SUBLAKE.

1886—1896.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Elevation of surface	zero	1".0	12".0	32".3	49".1	59".2	63".4	56".5	41".7	26".6	11".9	5".9

Since the table was built up from *averages* we must not expect it to emphasize special variations; for the grouping of averages resembles the grouping of pictures in composite photography. The combination invariably brings out *class likenesses to the exclusion of individual features.* Thus the table loses sight of an extraordinary year, like 1889—full of plus quantities—also seasons of drought, like 1894 and 1895. It, however, clearly shows that influx has a tendency to prevail between February and July, inclusive, and efflux to hold the mastery during the remaining months of the year.

Experience proves that these limits are subject to variations, also that influx period usually lasts five *months* from the day it commences. Thus:

When the influx begins in			January	it culminates in	June.
"	"	"	February	"	July.
"	"	"	March	"	August.
"	"	"	April	"	September.

It occasionally happens that the volume of influx *exactly equals* that of efflux. At such times the sublake remains stationary; thus:

1892—June 13 to July 23,	Sublake immovable for		40	days.
1893—February 4 to March 12,	"	"	36	"
1895—January 1 to March 7,	"	"	65	"

In all like cases the true turning-point is located midway in the stationary period.

Close observation shows that a copious rainfall can make its journey to the sublake in twenty-five days, provided the soil has been *saturated* by frequent storms. But when the ground is moder-

ately dry it takes sixty days to cover the same distance. A period of drought prolongs the time to at least ninety days. Just in proportion, then, as the dryness of the soil increases will the time of descent be prolonged.

For the present we shall accept as facts the following conditions, viz.: that the ratio of water volume at Hill Crest to the volume of rock holding the water is as 1 : 66. That every inch in depth of the well holds 11.1 gallons. Also that the normal flow per square foot of water-bearing surface is 14.2 gallons in twenty-four hours. The proof will appear later on.

Efflux.

It has been found by experiment that efflux escapes over the border of the sublake very much like *water escapes through a weir,* and that for a given area of discharge the volume of flow is contracted to 60 per cent. of the normal, it therefore amounts to about 8.6 gallons per square foot in twenty-four hours.

Since every inch in depth of well holds 11.1 gallons, the depth equivalent to this flow will be $\frac{8.6}{11.1} = 0.78$ inch; in other words:

$$\underline{\textbf{Daily Efflux} = \textbf{0.78 inch,}}$$

or

$$\underline{\mathbf{E = 0.78}} \text{ (expressed in depth of well).}$$

If we examine the records for 1890 and 1891 (when the ground was thoroughly moistened and the flow uniform) we find good illustrations of efflux extending over long periods.

In 1890 sublake lowered	108 inches in	138 days	(June 20 to Nov. 5),
" 1891 " "	72 " "	92 "	(June 20 to Sept. 20),
Total,	180 inches in	230 days,	

which gives an average

$$\underline{\textbf{Daily fall of sublake}} = \frac{180}{230}$$

or

$$\underline{\mathbf{E = 0.78}}$$

Showing complete accord between the data of calculation and those of experiment. This knowledge of the daily flow enables us to determine the yearly, which amounts to

$$\underline{0.78 \times 365} = \underline{284.7 \text{ inches in rock depth.}}$$

Since there are 66 cubic inches of porous rock around Hill Crest well to every cubic inch of water in suspension, the rock must contain as many inches of the annual rainfall as 66 is contained times in 284.7 inches. Therefore

$$\underline{\textbf{Annual Efflux}} = \underline{\textbf{4.32 inches}} = \underline{\textbf{9 \% rainfall.}}$$

or

$$\underline{\textbf{Monthly Efflux}} = \underline{\textbf{0.36 of an inch.}}$$

Experiment shows that annual fluctuations of the lake do not ultimately destroy the normal height; consequently efflux is an exact measure of that portion of the rainfall which, overcoming all hindrances, arrives in safety at the sublake.

Influx.

Influx descends at every point over the surface of the sublake and at times greatly exceeds efflux, for the latter only takes place around the border. *The total influx is always equal to efflux plus or minus a factor determined by observation.*

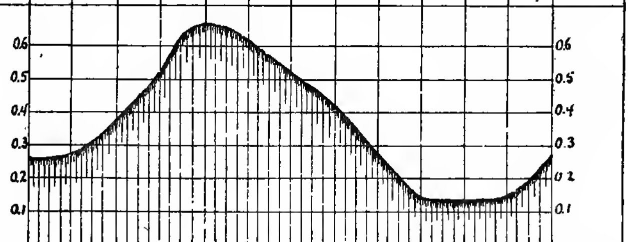
Since rainfall and efflux are expressed in inches of solid water, it is necessary to consider the water of influx distinct from the soil or rock into which it has flowed, and express it also in inches of solid water. This can be done for Hill Crest by dividing the rock depths by 66; the quotients will be the solid water equivalents. Calculating the same for Table I. we have:

January . . .	5.9″	diffused	water	=	0.09″	solid	water.
February . . .	1.0	"	"	=	0.015	"	"
March . . .	11.0	"	"	=	0.167	"	"
April	20.3	"	"	=	0.308	"	"
May	16.8	"	"	=	0.254	"	"

and so on. Expressing influx in tabular form we have

TABLE III.—TOTAL INFLUX.

1886.—1896.	JAN	FEB	MAR.	APR	MAY.	JUN.	JULY	AUG.	SEP	OCT	NOV.	DEC	TOTAL
MONTHLY AVERAGES.	-.09	.015	.167	.308	.254	.154	.064	-.105	-.224	-.229	-.223	-.091	EQUAL
EFFLUX.	.36	.36	.36	.36	.36	.36	.36	.36	.36	.36	.36	.36	4.32"
TOTAL INFLUX	0.27	.375	.527	.668	.614	.514	.424	.255	.136	.131	.137	.269	4.32"
PERCENTAGE.	6.25	8.68	12.20	15.46	14.21	11.90	9.80	5.90	3.15	3.03	3.17	6.25	100%



A moment's glance at these results shows that the least amount of water reaches the sublake during the month of October, and the greatest during April. These two months—separated by five intervening months of winter and five of summer—are characterized as months in which the average temperatures of *atmosphere, earth, and sublake* are practically *equal* (viz., 53° F.). As regards temperature, we should note, in passing, that in general terms :

$$\left\{\begin{array}{c}\text{The daily temperature}\\ \text{of ground water.}\end{array}\right\} = \left\{\begin{array}{c}\text{The average temperature of the}\\ \text{atmosphere throughout the year.}\end{array}\right\}$$

From this general consideration of the questions of influx and efflux, we enter more particularly into that of distribution.

RAINFALL.

The first step in our study of rain distribution is naturally a settlement of the question of monthly rainfall.

Fortunately, the Annual Reports of the Philadelphia Water Department furnish complete data for the Neshaminy—a small stream which empties into the Delaware. This stream drains a water-shed

of 140 square miles. The country through which it passes resembles that around Bryn Mawr, and the annual rainfall is the same in both places.

Collating said data, we have:

TABLE IV.—RAINFALL AVERAGES.

1886.-1896.	JAN	FEB.	MAR.	APR.	MAY	JUN	JULY	AUG	SEP.	OCT	NOV.	DEC.	TOTAL
RAINFALL	4.08	3.80	3.78	3.46	4.95	4.20	5.31	4.47	4.18	3.54	3.72	3.24	48.73

PERCENTAGE	8.	8.	8.	7.	10.	9.	11.	9.	8.	7.	8.	7.	100%

If we begin with the middle of April and divide the year into halves, we find:

Rainfall of the summer half = 26.61 inches,
" " winter " = 22.12 "

Difference 4 49 inches,

showing a difference of 20 per cent. more rain in summer than in winter.

SURFACE-FLOW.

The next question is: What amount of rainfall reaches the stream by passing directly *over* the surface of the ground? In answer, we note first that the Neshaminy has been found to average as follows:

TABLE V.—STREAM-FLOW.

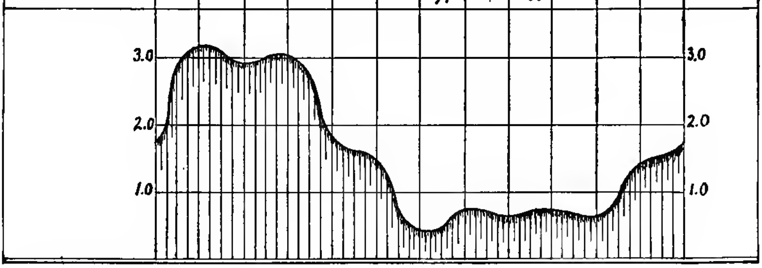
1886—1896.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug	Sept.	Oct.	Nov.	Dec.	Total.
Stream-flow	3.54	3.22	3.39	2.18	1.89	0.78	1.13	0.98	1.13	0.99	1.78	2.12	23 13

Whence it appears that the maximum amount of rainfall reaches the Neshaminy in January and the minimum in the month of June.

The stream-flow, however, includes *all* those waters of efflux which the sublake is constantly pouring into the stream. If, now, we separate efflux from stream-flow we shall determine what amount passes *over* the surface of the ground.

TABLE VI.—SURFACE-FLOW.

1886.-1896.	JAN.	FEB.	MAR.	APR.	MAY	JUN.	JULY	AUG.	SEP.	OCT.	NOV.	DEC.	TOTAL.
STREAM FLOW.	3.54"	3.22"	3.39"	2.18"	1.89"	0.78"	1.13"	0.98"	1.13"	0.99"	1.78"	2.12"	23.13"
EFFLUX.	.36	.36	.36	.36	.36	.36	.36	.36	.36	.36	.36	.36	4.32
SURFACE FLOW	3.18	2.86	3.03	1.82	1.53	0.42	0.77	0.62	0.77	0.63	1.42	1.76	18.81"



Since the annual rainfall equals 48.73 inches, we have:

$$\text{Stream-flow} = \frac{23.13}{48.73} = 48 \text{ per cent. of rainfall.}$$

$$\text{But we found efflux} = 9 \quad \text{"} \quad \text{"}$$

$$\therefore \quad \text{Surface-flow} = 39 \text{ per cent. of rainfall.}$$

ANNUAL FLOW.

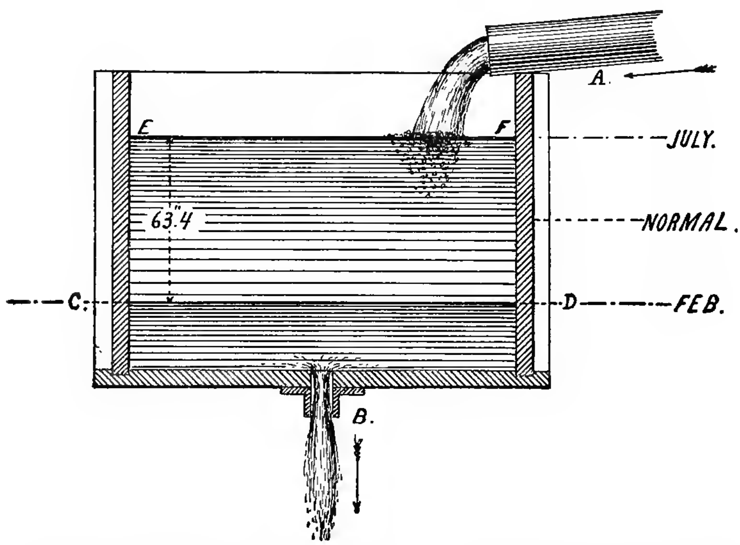
The average height of water in the ground for each month of the past ten years is found in Table II. and Fig. 2. These dimensions clearly indicate that (beginning with the month of February) the water steadily rises, as if a wave was in progress, and gains in volume each month until August, by which time it attains a height of 63.4 inches; after this it begins to subside, and finally reaches the old level in January.

Referring to Table III. we perceive that influx begins in October with 0.13 of an inch; that it culminates in April with a height of 0.67 of an inch, and subsequently drops back to its original level by the 1st of October. From which it appears that although the lake receives its maximum influx by the end of April, the wave attains its greatest elevation by the end of July. Here, then, is a difference of three months!

To what is this great difference due? We answer, mainly to the fact that efflux is *constant* and refuses to carry off (before the proper time) any excess that may be brought by influx. It follows that all such excess steadily accumulates. At last, when influx becomes less than efflux, subsidence ensues.

We can illustrate this rise and fall of the water by Fig. 4. Let ***E C D F*** represent the section of a tank, having an outlet-pipe

FIG. 4.



at ***B*** which is left open *all* the time. The inlet-pipe ***A*** is of greater diameter than ***B***. Turn on water and fill the tank to the level, ***C D***. If now we graduate the flow at ***A*** so that influx and efflux are equal, the water-level will remain unchanged at ***C D***. But if we open ***A***

wider and admit a greater volume the surface, *C D*, will steadily rise to say *E F*. Now partly close *A*, so that efflux becomes greater than influx, and immediately the surface, *E F*, will recede toward *C D*, thus giving in miniature all of the events of that wave-like flow which each year takes place at the sublake. If we represent the annual efflux by 100 per cent., we have for

$$\text{One month, } \frac{100}{12} = 8\tfrac{1}{3} \text{ per cent.}$$

When, therefore, influx exceeds $8\frac{1}{3}$ per cent. the lake will rise, and when it drops below $8\frac{1}{3}$ per cent. the lake will fall. Subtracting this percentage of efflux from the percentages of influx, given in Table III., we find the percentage of accumulation for each month in the year

TABLE VII.—PERCENTAGE OF ACCUMULATION.

1886—1896	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total
Influx	6.25	8.68	12.20	15.46	14.21	11.90	9.80	5.90	3.15	3.03	3.17	6.25	+100
Efflux	-8.33	-8.33	-8.34	-8.33	-8.33	-8.34	-8.33	-8.33	-8.34	-8.33	-8.33	-8.34	-100
Accumulation,	-2.08	0 35	3 86	7.13	5.88	3.56	1.47	-2.43	-5.19	-5 30	-5.16	-2.09	0

Gathering the positive and negative percentages into separate columns, we have:

Plus or Rising Percentage.		Minus or Falling Percentage.	
February,	+ 0.35	August,	— 2.43
March,	3.86	September,	— 5.19
April,	7.13 = Max.	October,	— 5.30 = Min.
May,	5.88	November,	— 5.16
June,	3.56	December,	— 2.09
July,	1.47	January,	— 2.08
Total,	+ $22\frac{25}{100}$ %	Total,	— $22\frac{25}{100}$ %

Which proves that, beginning with February, the lake cannot help rising, because the influx accumulates steadily until August, after which time efflux gains the mastery and eventually works off the entire surplus.

It should be noted that the accumulation averages $22\frac{25}{100}$ per cent. of the water that annually reaches the sublake, or

$$0.2225 \times 284''.7 = 63.4 \text{ inches},$$

the same maximum as given in Table II. and Fig. 2. We therefore regard the annual rise of water in the well as a natural sequence to *accelerated influx coupled with constant efflux* In like manner the annual fall is due to *retarded* influx coupled with constant efflux.

On a grand scale this phenomenon finds its counterpart in the annual rise and fall of the waters of the great lakes. The outlets—the St. Mary, the St. Clair, and the Niagara Rivers—are relatively small. The average summer rainfall (as observed at Milwaukee, Wis., by the State Weather Service) exceeds the winter downpour by more than 40 per cent. This excess causes the surface of

Lake Superior	to rise	13	inches	between March	and September.
Lake Michigan	"	12	"	" January	and July.
Lake Erie	"	15	"	" February	and June.

These elevations are based on measurements taken by the Engineer Corps of the United States Army [1860 to 1887], and show precisely to what extent the restricted outlets bank up the water in the respective lakes.

We also notice the working of the same principle during summer days. The hottest part does not occur at the noon hour—when the sun is on the meridian—but several hours later in the afternoon. In this case the accessions of heat arrive more rapidly than radiation is able to carry off. Radiation, however, keeps on apace, and, at last attaining the mastery, temperature falls. Ice-caves furnish still another example of the gradual procession in the seasons.

Sublake Evaporation.

Under the head of efflux we found that a constant quantity (0.36 of an inch of rainfall per month) passed away to the brook or stream quite independently of those (+ and —) quantities that cause a rise or fall in the surface of the sublake. Let us now consider more closely the (+ and —) quantities. In the first place, we see that

their sums are *equal*, because Table I. covers a cycle of time. Had either sum been in excess, the surface of the sublake would have stood accordingly higher or lower at the end of the period. Next we notice that the quantities have a minimum amount developed in the month of October. Searching for the cause of this minimum we find it in the intense heat, great evaporation, and luxuriant vegetation incident to the month of July, which parch the soil and develop evaporation *over the surface* of the sublake. The effect is cumulative, but maturity is reached after an interval of three months. Remembering that the maximum supply reached the lake in April and did not cease to manifest itself until after July, we conclude that in order to compare events happening at the surface of the sublake, with those at the surface of the ground, we must set our series of monthly averages (given in Table III.) *backward* three months and cause it to read thus:

TABLE VIII.—SUPPLY FOR SUBLAKE EVAPORATION.

1886—1896.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug	Sept.	Oct.	Nov.	Dec.	Total.
Sublake:													
Increase	.308″	.254″	.154″	.064″							.015″	.167″	+0.96″
Evaporation					-.105″	-.224″	-.229″	-.223″	-.091″	-.09″			-0.96″

We see, then, that 0.96 of an inch is virtually set aside from the general supply to meet the requirements of evaporation which takes place at the sublake's surface, and that the increments composing this 2 per cent. of rainfall may be considered a sort of reserve fund from which the evaporations make draft without affecting the other quantities. Their balance, therefore, determines the rise or fall of the sublake.

Let us now compare the results secured in Tables VI., III. and II., and determine the successive stages of rainflow. We discover at once that the ground receives or stores the greatest amount of water in the month of January (Table VI.), that this supply reaches the sublake in April (Table III.), and that the effect of accumulation wears off by the end of July (Table II.). In view of this six-

months' period we conclude that the stages of rainflow are marked by *very gradual progression,* and observation proves that sudden storms at the surface of the ground do not indicate corresponding movements of the sublake.

Surface Evaporation.

We have at length reached a stage in our analysis where satisfactory data can be determined for the following subjects:

1st. Monthly rainfall.
2d. Surface-flow to the river.
3d. Efflux to the river.
4th. Supply for sublake evaporation.

If the data for subjects 2, 3, and 4 be added together and their sum subtracted from subject 1, the remainder will represent that part of the rainfall which is taken up by evaporation and vegetation combined. Since vegetation withers in November and does not revive until April, we shall consider the months of December, January, February, and March by themselves, for in their case the differences will represent evaporation *alone.*

Making the subtraction as indicated, we have:

Table IX.

1886—1896.	Dec.	Jan.	Feb.	March.	Total.
Evaporation	0''.953	0''.232	0''.326	0''.236	1.75 inches

The excess in February is due to a mild interval that occurs in that month. During the remaining months of the year we have:

Table X.

1886—1896.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Total.
Evaporation and Vegetation	1''.216	3''.06	3''.42	4''.18	3''.49	3''.05	2''.55	1''.925	22''.89

making a total in the twelve months of 24.64 inches. If this amount were deposited in an open reservoir, 85 per cent. would pass off by evaporation. It, however, falls on the earth, and for some time remains within reach of the sun's rays, its downward progress being impeded by grasses, leaves, mosses, roots, etc. In view of the small amount of water that ultimately reaches the lake, it is fair to assume that the total evaporation will not exceed *one-half* the open reservoir amount—say 10.4 inches. But as 1.8 inches are evaporated from December to March, 8.6 will be evaporated from April to November. Subdividing and interpolating this quantity (having due regard to the character of the seasons) we obtain the following figures for evaporation:

TABLE XI.—SURFACE EVAPORATION.

1886—1896.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total.
Surface Evaporation	0.232	0.326	0.236	0.416	0.86	1.12	1.38	1 39	1.25	1.15	1.025	0.953	10.34

VEGETATION.

The amount of water taken up by vegetation can now be determined by subtracting the evaporation given in Table XI. (April to Nov.) from the joint supply of Table X.; we have

TABLE XII.

1886—1896.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Total.
Vegetation	.8	2.2	2.3	2.7	2.2	1.8	1.4	.9	14.3

We would add that the calorific effect of the seasons may be illustrated by the melting of ice under *uniform* conditions of exposure, as, for instance, ice that is kept in a refrigerator, thus:

Yearly average.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Ice melted, lbs.	300	300	400	600	750	900	1050	1050	900	700	600	450

which shows a melting power in July $3\frac{1}{2}$ times as great as that in February. It will be seen that our distribution gives a ratio of about 3 to 1, which closely approximates the actual.

Rainfall Distribution.

We have now accomplished our purpose of supplying the missing chapter in the history of rainfall and traced its flow both *over* and *within* the surface of the earth. It only remains to gather all results into one group so that relative proportions may become more apparent.

Table XIII.—Rainflow.

1886—1896.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total.	%
Surface flow	3.18	2.86	3.03	1.82	1.53	0.42	0.77	0.62	0.77	0.63	1.42	1.76	18.81	39
" Evaporation	.232	.326	.236	.416	.86	1.12	1.38	1.39	1.25	1.15	1.025	.953	10.34	21
Vegetation				.8	2.2	2.3	2.7	2.2	1.8	1.14	.9		14 30	29
Sublake increase	.308	.254	.154	.064							.015	.167		
" Evaporation					-.105	-.224	-.229	-.223	-.091	-.09			0.96	2
Efflux to River	.36	.36	36	.36	.36	.36	.36	.36	.36	.36	.36	.36	4.32	9
Monthly rainfall	4.08	3.80	3.78	3.46	4.95	4.20	5.21	4.57	4.18	3.54	3.72	3.24	48.73	100
Percentages Rainfall	8	8	8	7	10	9	11	9	8	7	8	7	100	%

Omitting details, the final results may be summarized as follows:

18 8 inches	flow away over the surface of the ground	= 39 %
10.3 "	pass off by evaporation	= 21
14.3 "	are taken up by vegetation	= 29
1.0 inch	is evaporated from surface of sublake .	= 2
4.3 inches	overflow from sublake to the river . .	= 9
Total 48.7 inches.		100 %

A rainflow table can be constructed for any other locality by closely observing the principles enunciated in the foregoing pages and by gathering data in like manner.

LAWS OF RAINFLOW.

SINCE our study of the phenomena of subterraneau flow has been made principally in the light of data supplied by Hill Crest well, it is proper as introductory to the study of the laws of flow, first to inquire into the physical characteristics of that well.

The curb is located 430 feet above sea-level and 80 feet above the nearest brook. The latter traverses a valley 1300 feet distant. The well was dug 6 feet in diameter to a depth of 65 feet through decomposed mica-schist, whose texture was sufficiently porous to admit the blade of a penknife. When first dug the water rose to a depth of 11 feet. The temperature of the water varied between 52° in winter and 54° in summer. Rock samples from this well were submitted to a skilled physicist, whose delicate experiments demonstrated the following facts:

Weight of 1 cubic foot of perfectly dry rock . . .	= 163.12 lbs.
" 1 " " of water-soaked rock (drained) .	= 174.675 "
" 1 " " " " " (not drained)	= 175.62 "

These figures show that *each cubic foot* of dry rock requires 11.56 pounds of water to fully moisten it, and when these demands are satisfied it further possesses a storage capacity of 0.945 pound of water subject to call. As one cubic foot of water weighs 62.3 pounds, the interstices of the rock will approximate $1\frac{1}{2}$ per cent. of its volume, which gives us the constant relation of 1 volume of water to 65 volumes of rock, or

$$R = \frac{62.3 - 0.945}{0.945}$$

or

$$\mathbf{R = 65.}$$

The well itself is lined with loose stones to within 10 feet of the top. The rest of the way the stones are laid in cement, and an extra

foot added to the wall. On it the curb was laid in a bed of cement, and the surrounding ground so filled in, rammed, and rounded as to make it utterly impossible for any surface-water to effect an entrance. A float was arranged in the well, with a suitable scale at the curb for indicating the depth of water. It may be objected that a 6-foot well is not an accurate unit of measure; that but few wells are true cylinders; that the diameter varies at different points; also that the displacement due to the stone lining is an unknown quantity! Granted that such irregularities do occur, but they only affect the sectional area of the well, and, whenever facilities exist for determining its average water-section, they constitute no valid objection. The section can be ascertained by pumping, say, 500 gallons into a tank, recording how much the lake falls and the tank fills. The respective depths will be inversely as the sections, and we have:

$$\text{True water section} = \frac{\text{Section of tank} \times \text{Rise in tank}}{\text{Fall of Lake.}}$$

For Hill Crest the true water-section amounts to an area of 17.8 square feet. Hence,

Every inch in depth represents 11.1 gallons.

Since the stone lining stands free of the earth's surface, the influx of water will not be impeded, and the entire circumference may be counted as effective water-bearing surface. Hence,

Every inch in depth will have 1.57 sq. ft. water-bearing surface.

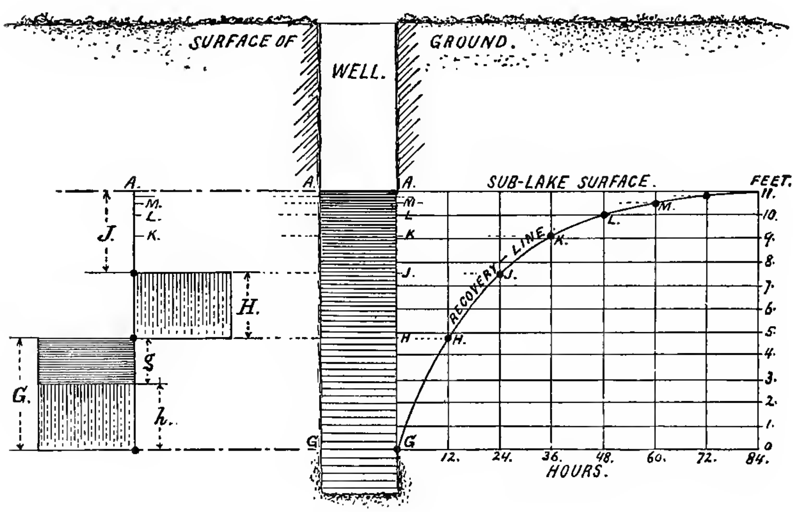
Fading-Flow.

The flow of water through a porous soil differs greatly from its passage through ordinary pipes. In the case of the latter the friction is relatively small, but in the former it is very great. The soil not only resists the progress of the water at every point, but sustains it so that head has no effect, and at the outlet the water escapes drop by drop from numberless pores. So slow is this process that each square foot of final area yields at Hill Crest only a few gallons every twenty-four hours.

The best method to study flow is to pump the water from a well

and make record of the quantities and times of arrival during various stages of recovery. The intervals between readings must be regulated by the freedom with which the water returns. The data thus secured can be plotted as in Fig. 5, and the recovery line developed. Such a diagram will give the requisite material for calculating flow from every square foot of exposed surface. When the water-level stands at ***G***, the water in the ground will flow from the cylindrical surface, ***AG***. The influx will cause the level to rise from

Fig. 5.



G to ***H*** in a certain number of hours, and thereby reduce the area of the feeding surface from ***AG*** to ***AH***. The mean depth of the feeding surface during the influx from ***G*** to ***H*** will equal ***AH*** $+ X \times$ ***GH***. In like manner the mean depth for a rise from ***H*** to ***J*** will equal ***AJ*** $+ X \times$ ***HJ***, and so on for other volumes.

In order to discover the values of the unknown factor X, we must carefully compare the volumes of flow which take place in two successive strata within equal lengths of time. Take, for instance, the strata G and H, as shown on the left-hand side of Fig. 5. In the first place we observe they represent two flows of entirely different character. The flow from G stratum is a *fading flow*, because the

outpour encroaches on the flowing surface and diminishes its area until at length the surface, G, is *entirely* submerged. The flow from H stratum meantime is, on the contrary, a *uniform flow*, because its outpour falls into the cavity G and leaves the H flowing undisturbed.

We shall aim, therefore, to compare *fading flow* with *uniform flow* and thus further our search for the unknown factor X.

We learn from experiments recorded in Table XIV. that the

Stratas J.H.G.	furnished	633	gallons	in 12	hours.
" J.H.	"	366	"	12	"
	Difference =	267 gallons.			

Or, more concisely,

$$\begin{array}{l} \text{The flow from G} = \text{h} + \text{g} = 1.00 \\ \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{H} = \text{h} \qquad = \ .58 \\ \hline \qquad\qquad \text{Difference} = 0.42 \text{ of G} = \text{g}. \end{array}$$

In this stage of the inquiry it is important to note, that when the G flow finally fades away, the H flow then ceases to be a *uniform one*, and in turn becomes, like G, a *fading flow*, which submerges H in twelve hours. But this distance H is a counterpart of h in the flow G. It follows that g (the remainder) must have come from H itself while it was running as a *uniform flow*.

In other words:

$$\left\{ \begin{array}{c} \text{H stratum} \\ \text{(with uniform flow).} \end{array} \right\} = 0.42 \text{ of } \left\{ \begin{array}{c} \text{G stratum} \\ \text{(with fading flow).} \end{array} \right\}$$

Now G stratum by fading flow was covered in twelve hours. Therefore to cover the same surface by *uniform flow* would take as many times 12 as 0.42 is contained times in unity $= \frac{1.00}{.42} = 2.38$, and $12 \times 2.38 = 28.57$ hours. From which it is evident that *uniform flow* and *fading flow* are inversely as their times of flow.

So that:

$$\text{G uniform : G fading :: 12 : 28.57}$$

and

$$\text{G uniform} = \frac{12}{28.57} \text{ G fading}$$

or

$$\text{G uniform} = 0.42 \ \text{ G fading.}$$

But G uniform is the same as flow from a mean area, and G fading the same as flow from a submerging surface.

Hence, we have in general :

Mean area of a submerging flow = 0.42 of the submerged surface.

Consequently,

$$\chi = 0.42$$

So that 0.42 is the unknown factor, for which we have been searching.

Applying same line of reasoning to other strata, we have :

Water-bearing surface for $HG = AH + 0.42\ HG$
" " " " $JH = AJ + 0.44\ JH$
" " " " $KJ = AK + 0.43\ KJ$ and so on.

Having thus determined the respective water-bearing surfaces, we finally divide each volume of recovery, ***GH, HJ,*** etc. (expressed in gallons), by the area of its water-bearing surface (expressed in square feet), and the quotients will measure the amount delivered by each square foot of water-bearing surface in the respective intervals of time.

Experimental Results.

A series of experiments was conducted at Hill Crest when the lake was in a state of repose. The pump was applied and 1468 gallons of water removed, which lowered the level $132\frac{1}{4}$ inches, and furnished during recovery material for the following table:

Experimental quantities are given in the first two columns. Observations were made at twelve hour intervals and each successive rise of surface was measured by inches. Dimensions and quantities given in the remaining columns were determined by calculation according to principles already explained. A little later, when speaking of area of flow, we shall direct attention to the fact that, as recovery amounted to only 131 inches, it did not fully restore what pumping removed, but caused a shrinkage of $1\frac{1}{4}$ inches.

TABLE XIV.—EXPERIMENT OF FEBRUARY, 1893.

Recovery.			Mean depths.					Yield per sq. ft.		Percentage.		
Hours.	Rise.	Gallons.	Values of χ	Depth of Free surface.	Mean of fading surface.	Total depth.	Water-bearing surface.	12 hrs.	24 hrs.	Flow.	Vortex.	Vortex depression.
	″			″	″	″	Sq. ft.	Gals.	Gals.			″
12	57	633	0.42	74	24	98	154	4.11	8.2			30
12	33	366	0.44	41	14 5	55.5	87	4.2	8.4	60 %	40 %	16.4
12	18.5	205	0.43	22.5	8.0	30.5	48	4.27	8.6			9
12	10.5	117	0.4	12	4.2	16.2	25.4	4.6	9.2	65	35	4.2
12	6.5	72	0.45	5.5	2.9	8.4	13.2	5.45	10.9	77	23	1.3
12	3.5	39	0.43	2	1.5	3.5	5.5	7.1	14.2	100	0	0
12	2	22	0	0	1	1				100		
84	131″	1454		Average =		23 %	of total.		Mean	=75	%	

Among other facts, we learn from the above that every square foot of Hill Crest rock has a normal flow of 14.2 gallons in twenty-four hours, also that complete recovery takes place in 84 hours. In addition to the above, like experiments were made in the months of January, February, March, June, July, September, and December (but in different years), and for quantities varying between 1000 and 3000 gallons. As the data so obtained included every variety of condition and fully confirmed the results of the foregoing series, we may safely rely upon the representative character of the figures given in the table.

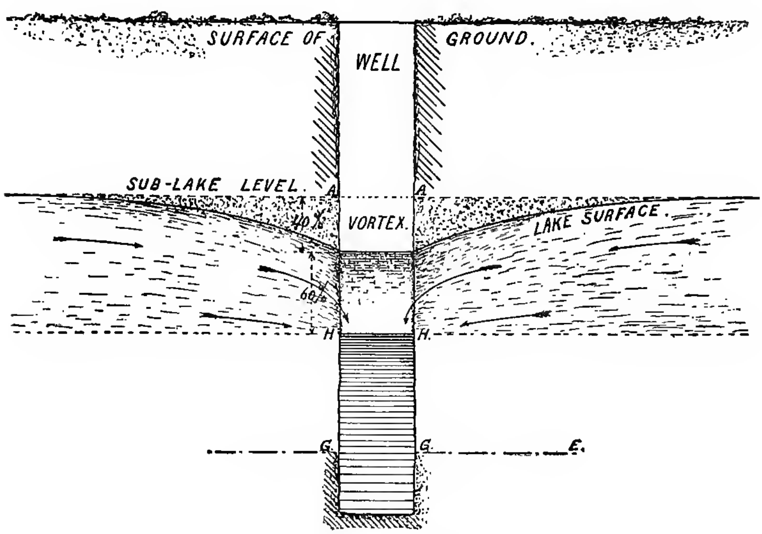
VORTEX-FLOW.

In the percentage column of Table XIV. we notice that the flow varied during the recovery period, the contraction reducing it to 60 per cent. of the normal—the *same as for water passing through a weir*—and that more than $\frac{8}{10}$ of the water returned on this basis.

After this the rate of flow expanded until finally the return entered at the normal rate, and by calculation the average for recovery was found to be 75 per cent. of the normal.

This variation in flow is illustrated by Fig. 6. When the well was emptied from ***A*** to ***G***, the water in the ground flowed from all directions horizontally toward the cavity ***AG***, and formed a depression or vortex around the inlet to the well, which, of course, proportionally diminished the effective area of discharge. When

FIG. 6.



the depth was $AH = 74$ inches, the vortex attained a depth of 30 inches. As fast as the water flowed into the well the reduction in area was distributed between vortex and water-bearing surface. This equality of distribution lasted for thirty-six hours. Subsequently the percentage of water-bearing surface increased and the vortex correspondingly diminished. Finally every square foot of water-bearing surface regained its normal yielding power, the vortex disappeared, and the water-level merged with the sublake level.

We shall now state results of experiments made during the years 1890–'93 and '97, so as to demonstrate what measure of harmony

existed under different conditions, with one standard porosity. In doing this, it will not be necessary to give every step with that minuteness which characterizes Table XIV., but only the salient points showing the true nature of the flow. By grouping the results, we obtain the following:

TABLE XV.—CONDENSED STATEMENT.

Date of experiment.	Gallons recovered.	Hours of recovery.	⅞ of entire Recovery had a flow of	Mean water-bearing surface.	Mean flow.
September, 1890 . . .	2770	84	60 pr.ct.	23 pr.ct.	75 pr.ct.
February, 1893 . . .	1454	84	60 "	23 "	75 "
February, 1897 . . .	822	84	60 "	23 "	75 "

We note that in *every case* complete recovery took place in 84 hours; although the amounts recovered varied between 800 and 2800 gallons, the *times did not vary.* Also that about $\frac{7}{8}$ of the entire recovery came with a constant flow of 60 per cent., leaving only $\frac{1}{8}$ for variable flow. Further, that the mean water-bearing surface was in each case 23 per cent. of the entire surface, and the mean flow was 75 per cent. of the maximum.

ROCK- AND WATER-SECTIONS.

The experiments described under the head of fading flow prove that for any given ratio (between the solid rock-section and that of the interstices through which the water percolates) recovery always takes place in a *fixed time,* and the hours do not vary from this standard, no matter how great the volume. Thus 3000 gallons will return in identically the *same* time as 1000 gallons. We also saw under the head of efflux that a *constant* flow characterized a fixed ratio of rock- and water-sections, so that no matter how great the supply, the rate of efflux did not vary. From which we learn that every locality has a characteristic:

Fixed time of recovery and a constant rate of flow.

For the purpose of establishing a formula expressive of the mutual relation existing between these conditions we shall adopt the following:

Notation.

$E =$ Max. daily efflux measured in inches of depth.
$E \times 11.1 =$ " " " " in gallons.
$F =$ " " flow in gallons, from every sq. ft. of water-bearing surface.
$H =$ Total hours required for recovery.
$R =$ Number units rock-section to each unit of water.
$R + 1 =$ Rock- and water-sections combined.

The value of E is determined by observations made on the descent of the sublake surface, as explained under the head of efflux. Dividing same by 24 we find the hourly descent. Taking one hour as our standard unit, we have:

$$\frac{(R+1)}{24} = \text{depth of saturated rock.}$$

$$\frac{E}{24} = \text{depth of water recovered;}$$

but these depths are equal,

$$\therefore \quad \frac{R+1}{24} = \frac{E}{24}.$$

It is, however, a fact that the flow producing the depth E does not cease when the height E has been reached, but will run for many hours before it spends itself, or all the interstices of the depth $\frac{R+1}{24}$ have emptied themselves, so as to restore the equilibrium.

The total value is therefore equal to

$$\frac{H \times E}{24}$$

and we have the relation of

$$\mathbf{R + 1 = E.H.}$$

or

$$\mathbf{R = (E \times H) - 1.} \quad . \quad . \quad . \quad (1)$$

The investigation of vortex-flow also furnished a means for expressing the relation between the terms E and F.

Thus:

$$60\ \%\ \mathbf{F} = \mathbf{E} \times 11.1.$$

or

$$\mathbf{F} = 18.5 \times \mathbf{E}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Example: For Hill Crest the efflux E = 0.78, and the total hours required for recovery H = 84; substituting these values, we have:

$$\mathbf{R} = (0.78 \times 84) - 1.$$

Hence

$$\mathbf{R} = 65 = \text{Number units rock-section to each unit of water.}$$

Shrinkage of Sublake.

When the surrounding country is quite level and the sublake area of great extent, the removal of 1000 gallons by pumping will produce no more impression upon the sublake than would result from taking a bucket of water out of a pond. Such effect, however, would not be the case when the sublake underlies an undulating country, for the elevation of the ground naturally limits the lake area, so that a depression (say, one or more inches) would result from pumping out 1000 gallons. In the following investigation this depression will be spoken of as inches of shrinkage.

To properly determine shrinkage involves a series of observations extending over a couple of weeks.

With the sublake in repose, as in Fig. 7, each observation for the first four days will have one and the same reading. On the fifth day the pumping takes place, and in consequence the lake falls D inches. Recovery follows on the succeeding days, until at last the return ceases. The sublake surface will now be found at C, short of its former level AB by a distance S. The readings taken on the next five days will show that the shrinkage S is permanent.

The *second case* of shrinkage is that where the sublake is steadily rising and should be plotted as shown in Fig. 8. Here the pumping interrupts the natural ascent of the water along the path AB.

The recovery ceases at the point C, a distance, CN, actually higher than the fifth-day reading, but falls short of the normal by a distance S.

The *third case* of shrinkage is developed when the sublake is steadily falling, and should be plotted as shown in Fig. 9. Here the total shrinkage seems equal to NC, but in reality BN is due

FIG. 7.

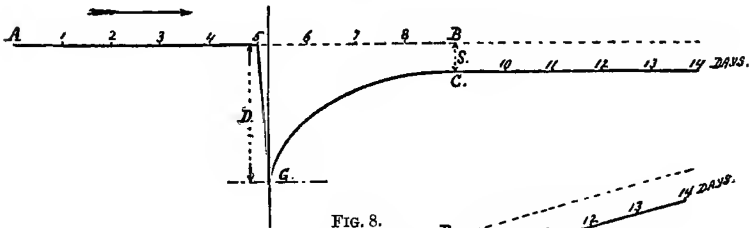


FIG. 8.

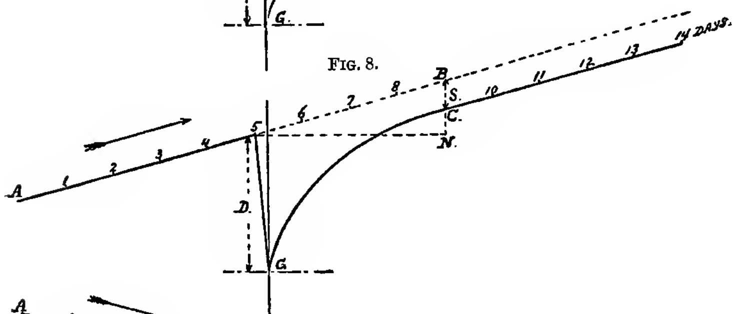
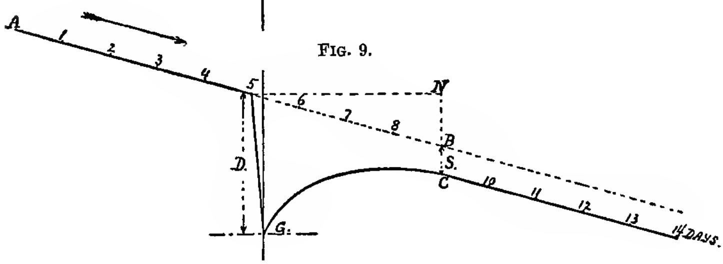


FIG. 9.



wholly to natural decline of the waters, while S is the shrinkage due to pumping. Whence we learn that the difference between readings (taken before and after recovery) *only* expresses true shrinkage when the lake is in repose. Also that this difference is of questionable magnitude when the sublake is steadily rising and *excessive* when the sublake is steadily falling. At times combinations of two cases occur and require careful consideration.

Velocity of Flow.

A soil or disintegrated rock may have a uniform texture, but if it is traversed by water-bearing fissures they will develop a more rapid flow than the discharge due to texture *per se*. For this reason one may calculate the absorbent power of a rock specimen without in reality determining the resultant flow, as the latter depends on the presence or absence of fissures. The Hill Crest experiments give evidence that its flow was due alone to porosity of soil. The well, therefore, is a good one for the purpose of investigating the question of velocity. But whatever may be the final result, we should consider it more in the light of an approximation than an exact measurement.

On general principles we have:

$$\left\{\begin{array}{c}\text{Original volume}\\ \text{occupied by}\\ \text{water and soil.}\end{array}\right\} = \left\{\begin{array}{c}\text{Mean area of}\\ \text{water-bearing}\\ \text{surface in sq. in.}\end{array}\right\} \times \left\{\begin{array}{c}\text{Velocity}\\ \text{per hour}\\ \text{in inches.}\end{array}\right\} \times \left\{\begin{array}{c}\text{Total}\\ \text{hours of}\\ \text{Recovery.}\end{array}\right\}$$

Notation.

D = Depth in inches of Q gallons.
H = Total hours required for recovery.
R = Number of units of rock-section to each unit of water.
$R + 1$ = Rock- and water-sections combined.
Q = Number of gallons in a well of D depth.
V = Average velocity of horizontal flow in inches per hour.

Embodying same in general formula, we have:

$$231\,Q\,(R + 1) = 144\,(0.23\,D \times 1.57 \times 0.75) \times V \times H \quad . \quad . \quad (3)$$

$$= 144\,(0.27\,D)\,V.H$$

$$\text{Since} \quad \frac{Q}{D} = 11.1$$

$$\mathbf{V} = \frac{\mathbf{67\,(R + 1)}}{\mathbf{H}} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

Example: In the case of Hill Crest $R = 65$ and $H = 84$. Substituting the values of R and H in equation No. 4, we have:

$$\mathbf{V} = \mathbf{52.6} \text{ inches per hour,}$$

which gives us the average velocity of flow in a horizontal direction during the time of recovery.

Area of Flow.

When a large quantity of water has been removed from a well by pumping, the question arises as to what area of sublake will be disturbed by the process of recovery. In other words, how far-reaching is the influence of the vacancy caused by pumping? The answer is that in some cases the area can be determined approximately, while in others it is practically unlimited, and those instances which show shrinkage at time of recovery are the only ones susceptible of calculation.

The estimate for area of flow can be made on the general principle of cubic volumes of soil, viz.:

$$\left\{\begin{array}{c}\text{Total vol. of soil}\\ \text{from which the}\\ \text{water is drawn.}\end{array}\right\} = \left\{\begin{array}{c}\text{That portion which}\\ \text{supplies inflow}\\ \text{during pumping.}\end{array}\right\} + \left\{\begin{array}{c}\text{That portion which}\\ \text{supplies inflow}\\ \text{during recovery.}\end{array}\right\}$$

Notation.

T = Total number of square feet in area of flow.

S = " " of inches sublake is lowered by shrinkage.

$\frac{h}{24}$ = Portion of a day occupied in pumping.

W = Influx (in gallons) during pumping.

Then:

$$\text{Total volume of soil from which the water is drawn} = T \times \frac{S}{12}$$

$$\text{That portion which supplies inflow during pumping} = \frac{231 \times W}{1728}(R+1)$$

But

$$W = \left\{\begin{array}{c}\text{Mean area}\\ \text{laid bare}\\ \text{by pumping.}\end{array}\right\} \times 60\,\% \left\{\begin{array}{c}\text{Daily}\\ \text{max. flow}\\ \text{per sq. ft.}\end{array}\right\} \times \left\{\begin{array}{c}\text{Time}\\ \text{of}\\ \text{pumping.}\end{array}\right\}$$

or

$$W = \frac{1.57\ D}{2} \times 0.6\ F \times \frac{h}{24}$$

Introducing the value of F from equation 2, we have:

$$W = 0.363\ D.E.h \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Substituting, we have:

$$\left\{\begin{array}{l}\text{That portion which}\\ \text{supplies inflow during pumping}\end{array}\right. = 0.0486\ D.E.h.(R+1).$$

And equation 2 expressed in cubic feet gives us:

$$\left\{\begin{array}{l}\text{That portion which}\\ \text{supplies inflow during recovery}\end{array}\right. = 0.27\ \mathrm{D}\ \frac{\mathrm{V}}{12}\ \mathrm{H}.$$

After assembling the terms we find:

$$\frac{\mathrm{TS}}{12} = 0.0486\ \mathrm{D.E.h.}\ (\mathrm{R}+1) + 0.27\ \mathrm{D.}\frac{\mathrm{V}}{12}\ \mathrm{H.}$$

Therefore:

$$\mathbf{T} = \frac{\mathbf{D}}{\mathbf{S}}\left[\mathbf{0.27\ V.H. + 0.58\ E.h.\ (R+1).}\right] \quad . \ . \ (6)$$

Example: In the case of the Hill Crest experiment of Feb. 1893,

D = $132\frac{1}{4}$	E = 0.78	S = 1.25
h = 3	H = 84	V = 52.6
	R = 65	

Required the area of flow, from which the well gathered its water at time of recovery?

It is only necessary to substitute the above values in equation 6, to find:

Area of flow = 135,680 square feet = $\mathbf{3\frac{1}{10}}$ Acres.

Although this investigation shows that in cases of shrinkage it is quite possible to determine the acreage covered by the sublake, still its contour line can never be fixed, for that depends wholly on the characteristics of the soil or bed-rock. For a given locality it may be either a circle, an oblong, or any irregular figure.

POPULAR MISAPPREHENSION.

Slow Penetration of Rainfall.

It is a great mistake to imagine that rainfall penetrates rapidly to the lake. This is rarely the case, and in many soils it takes months to accomplish the journey. Instances have occurred at Hill Crest wherein the ground-water steadily *lessened* in months of *heavy* rainfall; also instances in which the water steadily *rose* in times of severe drought.

For example, the surface of sublake *lowered* 52 inches during July and August, 1891, regardless of the fact that the rain fell in exceptionally large quantities, amounting to 10 inches. Again, the surface of the lake *rose* 42 inches between April 12 and May 3, 1891, notwithstanding the fact that *not a drop of rain* fell, while the water was rising!

These facts show that no investigator is able to predicate the condition of ground-water from the data of a rainfall record, nor can he use the latter for the former under any circumstances.

Typhoid Fever and Ground-Waters.

The theory of rainflow introduces a new view as to the healthfulness of ground-waters, when considered in their relation to the increase or decrease of typhoid fever.

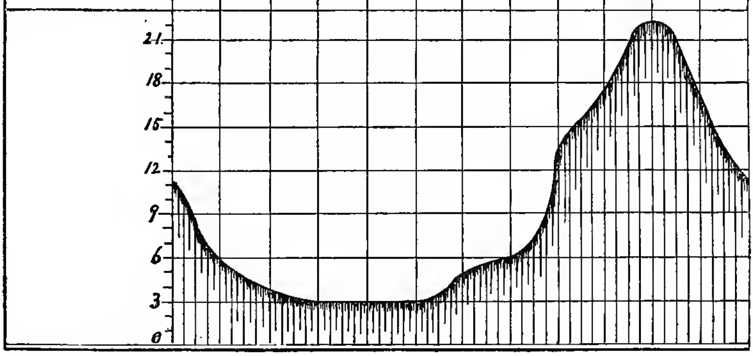
For more than thirty years the German theory has found many advocates. The leading idea has been that a very close relationship exists between the annual rise and fall of ground-water and the increase or the decrease of typhoid fever. The ratio being an inverse one, viz., as water subsides typhoid increases, as water rises typhoid diminishes. In our own country the subject has been carefully investigated by the Board of Health of the State of Michigan. (See Annual Report for fiscal year 1894.)

It is a notorious fact that many household wells are constructed with little regard to sanitary conditions. Some wells are exposed to

the air and sun, so that grasses and weeds grow during the summer months and fringe their border; while strong winds deposit dust and leaves over the surface of the water; also various forms of animal life enter and die. Then, too, many wells are concave at the mouth, so that surface-water finds ready entrance during heavy storms. Worse still, many wells are sunk in porous soils in close proximity to cesspools or leaky drain-pipes. As all like conditions can be discovered and remedied, such wells form no part of our investigation, but must be ruled out of the question. Examining the reports, we observe: *First*, that taking the health records given from 1889 to 1893 we are able to construct the following diagram:

TABLE XVI.—TYPHOID REPORTED IN MICHIGAN.

1889.–1893.	JAN.	FEB.	MAR.	APR.	MAY	JUN.	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
PERCENT OF TYPHOID.	6.	4.	3.	3.	3.	5.	6.	13.	17.	22.	17.	11.



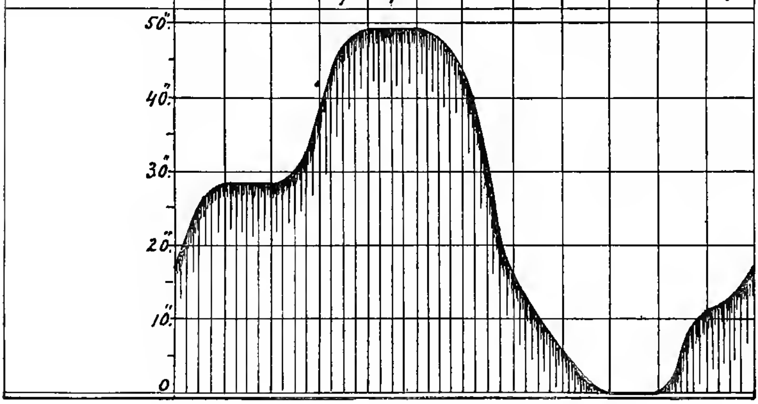
From which we learn that typhoid became alarming about the middle of August and attained its maximum virulence about the 1st of November; also that a small percentage existed during the first seven months in the year. *Second.* The annual rainfall in Michigan for the same period was about $\frac{2}{3}$ the rainfall in Philadelphia. *Third.* The "representative well" in the Capitol grounds at Lansing had an annual oscillation similar to that shown in Table II., only the zero occurred in February, and the July

elevation averaged 13 inches, the extremes being 11 and 24. Surely this record does not favor the ground-water theory because typhoid reached its worse stage three months *before* the lowest water-mark was touched, and the epidemic completely disappeared by the time that mark was reached. According to such a showing, subsidence would be chargeable with causing an epidemic, *both* to rage and to abate.

Let us now examine the data given in the same report for "*many wells scattered throughout the State of Michigan,*" whose waters were carefully watched between 1878 and 1883. We see at once that the average soil was more porous than that around the Lansing well, for the greatest rise in any one year was 96 inches and the least 40. Constructing a table like No. II. we find:

TABLE XVII.—AVERAGE RISE OF MICHIGAN SUBLAKE.

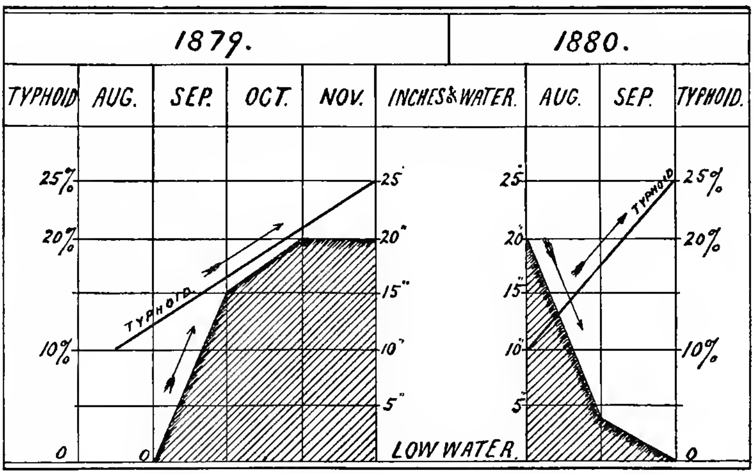
1878–1883.	JAN.	FEB.	MAR.	APR.	MAY	JUN.	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
LAKE RISE & FALL	28"	28"	38"	49"	49"	43"	16"	6"	0"	0"	11"	17"



At first glance this table seems to favor the popular maxim: high ground-water, little typhoid; low ground-water, typhoid an epidemic!

It must be remembered, however, that the process of averaging a six-years' record throws characteristic points into the shade and *only*

makes note of general features. For example, the 22 per cent. typhoid average came from a group whose extremes were 18 and 37; in like manner the May 1st high-water-day, had for extremes April 1st and July 1st. Whenever, therefore, we aim to establish a *true* relation between two events occurring in a given year, manifestly we must not vitiate our data by diluting it with the diverse records of five other years. But, on the contrary, we should strengthen it by finding two consecutive years which have characteristic features as near *alike* as possible, and use them as a basis for comparison. Let us take 1879 and 1880, because in those years the epidemics were of *equal* virulence; also the ground-water rose and fell almost *equal* distances. The most important events are given in the accompanying diagrams:



The ground-water stood in each year at its highest level May 1st. Typhoid attained the alarming stage of 10 per cent., say, August 1st to 15th, and the maximum amounted to 25 per cent. Now note that the typhoid in one case became alarming sixteen days *before* low water, and in the other sixty days. The difference of forty-five days shows that the *precise moment* of low water had nothing to do with the *origin* of the disease. Also note that in 1880 typhoid reached its most virulent stage on the *same day* as low water, but in

1879 it took *ninety days after* low water-mark was passed before it attained the same stage, showing that the precise moment of low water had *nothing* to do with the *development* of the disease. Incidentally the water rose in 1879, 20 inches. In harmony with the "low-water theory" such a rise ought to have put a decided check upon the disease, but nothing of the kind occurred. It in fact had no influence whatsoever! And why? Simply because these phenomena were not related to each other as cause is to effect. The two diagrams flatly contradict each other. In 1879 the epidemic must be credited to 20 inches of *rising* water, while in 1880 an epidemic of equal violence raged with 20 inches of *falling* water. We might compare other years, but would find like inequalities and divergences.

It is evident, therefore, that the Michigan data fail to establish any useful relation between the prevalence of typhoid and the height of the ground-water; whether we consider the data of the "representative well" at Lansing, or whether we take the figures given for the "many wells throughout the entire State," the result is one and the same.

According to the principles of rainflow, the lake is *not* a body of water that becomes more and more polluted as summer advances. Its surface is *not* lowered materially by evaporation; it does *not* change in temperature, nor is it productive of either animal or vegetable life. On the contrary, its surface is lowered by natural overflow; when fresh accessions arrive they come only through the superincumbent soil, so that every globule of the water is not only *perfectly* filtered, but in a rightly conditioned well it is both potable and healthful.

These considerations convince us that the typhoid ground-water theory is *not* supported by facts; also that whatever relation or synchronism does exist, it is merely a *coincidence*, and possesses no special significance.

Household Wells Should Never Fail.

Many property-owners have an idea they can secure a satisfactory well by digging far enough to find water; they then deepen the cavity, whatever may be necessary to hold the daily supply, and finally wall up the interior. The idea is an erroneous one and at variance with the principles of rainflow, for it

ignores the question of periodic fluctuations. The chances are that such a well will fail in times of great drought.

Fortunately the remedy is always at hand, and tedious observations are not necessary to solve the question. The right way is to search the neighborhood for some Resident of twenty or thirty years' settlement, whose well has *never failed,* and learn from him the *least depth of water* his experience can recall. If perchance that depth would be sufficient for your daily wants, measure the *present depth* of water in his well, and dig your own well deep enough *to secure precisely the same present depth.* For example: If the resident remembers one season in twenty years during which he had only 3 feet of water in his well, ask yourself the question: Would 3 feet as a minimum satisfy my wants? Afterwards measure his present supply. Suppose it amounts to 15 feet. Then dig your own well to whatever depth may be necessary to secure 15 feet of water. If, however, your wants exceed those of your neighbor, you should continue digging and deepen your own well enough to secure the excess also. Careful observance of this precaution is sure to give a never-failing supply.

RECAPITULATION.

It has been shown that a very large part of the annual rainfall passes away *over* the surface of the ground. Hence the importance of using every means to preserve our forests. For wherever the country is thickly wooded the undergrowth, ferns, leaves, and mosses arrest the flow of water, to the great benefit of the land; freshets seldom occur, also protracted periods of drought and failure of springs are scarcely known.

It has been demonstrated that the withdrawal of a thousand gallons from any well, located in a compact soil disturbs the sublake over many acres of ground. Therefore it is the part of wisdom always to anticipate the encroachments of a growing population by providing an independent water-supply, located far beyond the reach of contaminating influences.

The study has served the good purpose of clearing away certain popular misapprehensions with regard to the relations between rainfall and rainflow; between ground-water and typhoid fever; it has also suggested what precautions are necessary to insure a never-failing supply of health-giving water.

Our investigation has taught us to recognize the world-wide existence of the great subterranean lake, its characteristic features, its periodic fluctuations, and how to measure both volume and velocity of its unseen flow. The most striking features discovered are equations Nos. 1 and 2—showing the relation between rock- and water-sections; also the analogy existing between the laws of subterranean flow and the laws governing the discharge of water through a weir; the nature and process of recovery, likewise the relation between uniform flow and fading flow. Attention has also been directed to the fact that the *daily* temperature of ground water is equal to the *yearly* average temperature of the atmosphere.

We believe that the present research has developed an outline of the general laws of rainflow, which for the first time places the subject in its true light.